dinal and transverse coordinates; ν, kinematic viscosity; $\underline{a}$, thermal diffusivity; c_p, heat capacity at constant pressure; T_∞, temperature of the surrounding medium; I, impulse of the jet; Q, heat flux; ρ, density; d, nozzle diameter; ϕ, similarity variable; δ, thickness of the jet; f, stream function; θ, dimensionless temperature, $\theta = E(T - T_\infty)/TR_\infty^2$; Br, Brickman number, $Br = \mu u_0^2 d \exp(E/RT_\infty)/Q^*$, $Q^* = \pi\rho c_p u_0 d^2/4$; Sv, entrainment parameter of the fluid, $Sv = \mu\exp(E/RT_\infty)/\rho u_0 d$, $\beta = RT_\infty/E$; F, dimensionless heat flux, $F^* = \pi\rho c_p u_m \Delta T_m \delta^2$; Pr, Prandtl number, $Pr = \nu/a$; $Q_1 \sim (F - F_0)/d$; $Q_2 \sim \exp(F/(x_0 + \beta F)Sv)/x_B^2$.

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DETERMINING RHEOLOGICAL PARAMETERS FOR A DISPERSION SYSTEM BY ROTATIONAL VISCOMETRY

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An algorithm has been devised for inverse treatment in rotational viscometry subject to a priori uncertainty over the model. A model class has been formulated for rheologically stationary systems.

Dispersion systems are widely used, and determining their rheological characteristics is important in data support to optimum management.

Some methods of processing data from rotational rheometry [1-3] make inadequate use of the information from experiment to derive models and evaluate parameters. Also, simplified methods are usually used to evaluate rheological characteristics for nonlinear viscoplastic media [1, 3], and these may give substantial errors in inverse treatments.

One can process such data from an equation describing Couette flow in a gap between coaxial cylinders [1]:

$$\omega = \frac{1}{2}\int_a^\tau \frac{\dot\gamma(\xi)}{\xi}\,d\xi. \tag{1}$$

The relation between the stresses on the outer and inner cylinders is

$$a = \begin{cases} \alpha^2\tau, & \text{if } \tau \geqslant \tau_0/\alpha^2; \\ \tau_0, & \text{if } \tau \in]\tau_0, \tau_0/\alpha^2[, \end{cases}$$

where τ_0 is the dynamic shear stress (yield point), $\alpha = R_1/R_2$, and R_1 and R_2 are the radii of the inner and outer cylinders.

An inverse rheometric treatment involves choosing the state index $\hat\nu$ for the medium from a certain class ϑ of rheologically stationary models known a priori and then estimating the parameter vector $\hat p_\nu$ for that model. The ϑ class can be formed from the following models: Newtonian ($\nu = 1$) – $\dot\gamma = \tau/\mu$, Shvedov–Bingham ($\nu = 2$) – $\dot\gamma = (\tau-\tau_0)/\mu$, Ostwald ($\nu = 3$) – $\dot\gamma = (\tau/k)^{1/n}$, Herschel–Bulkley ($\nu = 4$) – $\dot\gamma = ((\tau-\tau_0)/k)^{1/n}$, Schulman–Casson ($\nu = 5$) – $\dot\gamma = (\tau^{1/n} - \tau_0^{1/n})^n/\mu$, etc. Here μ, τ_0, k, n are the rheological parameters.

Statistical methods are applied to treating the data [4], on the assumption that the discrepancy between the measurement vector τ having components $\{\tau_i\}$, $i \in \overline{1, N}$ and the theoreti-

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cal pattern $A(\omega, p_\nu)$ (subsequently, we write $A(p_\nu)$ for simplicity) is such that $\omega = \{\omega_i\}$ and is additive, so we can formalize the inverse treatment as

$$\tau = \begin{cases} \text{either } A(p_1) + n_1, \\ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \\ \text{or } \quad A(p_\nu) + n_\nu, \ \nu \in \vartheta. \\ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \end{cases} \tag{2}$$

Before we construct estimation procedures, we have to mention the distribution for the random discrepancies above. We assume that these discrepancies arise from measurement errors, and we assume a normal distribution. Practical interest attaches to cases when n_ν is a centered quantity having a covariance matrix C, which can be derived by experiment, and when the random component is centered, stationary, and independent at all the observation points. The latter particular case in general requires independent consideration in relation to operational rheological-parameter estimation from restricted data and the need for joint estimation of the random-component variance σ^2 in the covariance matrix $C = \sigma^2 I$.

We handle (2) with an algorithm [4] based on estimating the parameter vector $\hat{p}_\nu(\nu \in \vartheta)$ and then recognizing the state $\hat{\nu}$. We use maximum-likelihood estimators. The likelihood function is

$$L = \frac{1}{(2\pi)^{N/2} |C|^{1/2}} \exp\left[-\frac{1}{2}(\tau - A(p_\nu))^{\mathrm{T}} C^{-1} (\tau - A(p_\nu))\right], \tag{3}$$

where $|C|$ is the determinant for the covariance matrix ($|C| \neq 0$), C^{-1} is a matrix inverse to C, and $(\cdot)^{\mathrm{T}}$ is the transposed discrepancy vector. Then we have the following estimator procedures for the rheological-parameter vector:

$$\max_{p_\nu \in P_\nu} L(p_\nu) \Rightarrow \hat{p}_\nu, \ \nu \in \vartheta, \tag{4}$$

and model recognition

$$\max_{\nu \in \vartheta} L(\hat{p}_\nu) \Rightarrow \hat{\nu}. \tag{5}$$

On joint maximum-likelihood estimation for p_ν and σ^2 (case $C = \sigma^2 I$), the estimators $\hat{\sigma}_\nu^2$ and $\hat{p}_\nu$ are independent [4], and in essence the treatment does not differ from that of (4).

As matrix C is symmetrical and positive-definite, C^{-1} has a positive and symmetrical square root $C^{-1/2}$. Then (4) is equivalent to

$$\|C^{-1/2}(\tau - A(p_\nu))\| + F_p(p_\nu) \to \min, \ \nu \in \vartheta, \tag{6}$$

where $\|\cdot\|$ denotes the norm and

$$F_p(p_\nu) = \begin{cases} 0 \Rightarrow L_\nu = L_\nu, & \text{if } p_\nu \in P_\nu; \\ \infty \Rightarrow L_\nu = 0, & \text{if } p_\nu \notin P_\nu. \end{cases}$$

The functional F_p in (6) reflects the Lagrange principle for a treatment subject to constraints and in essence does not complicate the solution; the formulation is based on the specific class ϑ, in which each successive model extends the previous in some sense.

We first solve (1) for a nonlinear viscoplastic medium ($\nu \in \overline{4, 5}$). For states $\nu \in \overline{1, 3}$ (1) is readily solved analytically, and the solution has been given for example in [2]. Equation (1) is a Volterra-type integral one and can be solved from recurrent relations. One can use Newton–Coates quadrature formulas to approximate (1) to get a recurrent relation:

$$\tau^{j+1} = 12N_1\omega \left\{ \sum_{i=1,3}^{2N_1-1} [F(\tau_{i-1}^j) + 4F(\tau_i^j) + F(\tau_{i+1}^j)] \right\}^{-1} + a, \tag{7}$$

where $F(\cdot)$ is the expression in the integrand in (1), while $\tau^j{}_{2N_1}$ is approximation j for τ, $\tau^j{}_i = \tau^j{}_{i-1} + (\tau^j{}_{2N_1} - a)/2N_1$, and $\tau^j{}_{i-1} = a$ for $i = 1$.

Equation (7) converges uniquely to the solution for bounded τ on the set $D \in]a, +\infty[$ because the mapping of D into itself is continuous when the number of nodes $2N_1$ and the zeroth approximation τ^0 are chosen appropriately.

We construct the solution to (6) without constraints for a nonlinear viscoplastic medium ($\nu \in \overline{4, 5}$). For states $\nu \in \overline{1, 3}$, (6) can be solved analytically (for $\nu = 2$ with $\tau \geq \tau_0/\alpha^2$). We construct the algorithm on extremal classes [5].

Let p_ν^n be approximation n for the solution to (6) and let Λ be a diagonal matrix having elements λ that define the a priori standard deviations of the p_ν^n from $\hat{p}_\nu$. As the Fréchet derivative $A'(p_\nu)$ is continuous in the region of $p_\nu{}^n$, we first consider the following quasi-linear form in an auxiliary treatment:

$$\begin{cases} A'(p_\nu^n)\,h = \bar{\tau}, \\ \|\Lambda^{-1}h\| \to \min, \end{cases} \tag{8}$$

where $h = \hat{p}_\nu - p_\nu{}^n$ and $\bar{\tau}$ is some element for which one of the solutions to the first equation in (8) has the characteristic $A(p_\nu{}^n + h) = \tau$. The second equation in (8) defines the solution selection criterion.

As Λ is a linear and closed operator having the kernel Ker $\Lambda = \{0\}$, the extremal class for an operator A' linear in the region of $p_\nu{}^n$ is [5]

$$h = \Lambda^2 A'^*(p_\nu^n)\,\varphi, \tag{9}$$

where ϕ is an element from the definition region for the conjugate operator A'^* (* represents the Hilbert conjugation). Then (6) with (8) and (9) can be represented as

$$\|C^{-1/2}(A(p_\nu^n + \Lambda^2 A'^*(p_\nu^n)\,\varphi) - \tau)\| \to \min. \tag{10}$$

Kobrunov [5] proposed a convergent iteration for (10):

$$p_\nu^{n+1} = p_\nu^n + \alpha_n \Phi, \tag{11}$$

where $\Phi = \Lambda^2 A'^*(p_\nu{}^n)C^{-1}(A(p_\nu{}^n) - \tau)$, which enables one to obtain a solution when the relaxation parameter α_n is chosen from

$$\alpha_n = -\frac{\langle C^{-1}(A(p_\nu^n) - \tau)\,|\,A'(p_\nu^n)\,\Phi\rangle}{\langle C^{-1}A'(p_\nu^n)\,\Phi\,|\,A'(p_\nu^n)\,\Phi\rangle}, \tag{12}$$

where $\langle\cdot|\cdot\rangle$ denotes the scalar product. Consequently, the algorithm for solving nonlinear (6) without constraints amounts to choosing the zeroth approximation $p_\nu{}^0$ together with matrix Λ and implementing (11) on the basis of (12).

Then the algorithm for processing the viscometry data can be based on the following procedures.

1. One formulates class ϑ of possible rheological models.

2. C^{-1} is constructed from the measurements.

3. One solves (6) for class ϑ.

4. One puts $\hat{p}_\nu$ into correspondence with the set of possible solutions P_ν $(\nu \in \vartheta)$. If for some subclass $\vartheta_1 \in \vartheta$ one has $\hat{p}_\nu \notin P_\nu$, one can put $L(\hat{p}_\nu) = 0$ $\forall \nu \notin \vartheta_1$.

5. One calculates $L(\hat{p}_\nu)$ from (3) for the model class ϑ/ϑ_1.

6. One derives from (5) the model $\hat{\nu}$ best in the maximum-likelihood sense.

7. The performance in the solution is evaluated via $\hat{p}_\nu$ for model $\hat{\nu}$. The covariance matrix for the rheological-parameter vector can be obtained by inverting the Fisher information matrix [4]:

$$O = (A'(\hat{p}_\nu)\,C^{-1}A'^*(\hat{p}_\nu))^{-1}.$$

We consider processing data for a bentonite drilling solution as regards the interpretation provided by this method and some of the commoner ones. The measurements were made with a rheometer having $\alpha = 0.909$: $\omega = \{0.021;\ 4.189;\ 6.283;\ 8.378;\ 12.566;\ 20.944;\ 31.416;\ 41.888;\ 62.83\}$ sec^{-1}, $\tau = \{7.81;\ 15.62;\ 17.75;\ 18.82;\ 21.30;\ 25.56;\ 29.11;\ 33.01;\ 39.05\}$ Pa. The inverse treatment was handled with the $\vartheta \in \{2,\ 4,\ 5\}$ model class. To ensure identical processing conditions for the different methods, the covariance matrix was taken as $C = \sigma^2 I$. The best model was $\hat{\nu} = 5$ (Schulman–Casson), the rheological-parameter estimators being $\hat{\tau}_0 = 5.47$ Pa, $\hat{\mu} = 3.51 \cdot 10^{-3}$ Pa·sec, and $\hat{n} = 3.29$. The calculated shear stresses were $\hat{\tau} = \{7.52;\ 15.58;\ 17.36;\ 18.87;\ 21.39;\ 25.27;\ 29.29;\ 32.84;\ 39.33\}$ Pa.

The traditional method is based on approximate calculation of the shear rate gradient $\dot{\gamma} = 2\omega/(1 - \alpha^2)$, and the estimators are obtained by least squares as $\hat{\nu} = 5$, $\hat{\tau}_0 = 6.75$ Pa, $\hat{\mu} = 4.75 \cdot 10^{-3}$ Pa·sec, and $\hat{n} = 2$. The calculated shear stresses are $\hat{\tau} = \{8.53;\ 16.75;\ 18.54;\ 20.04;\ 22.62;\ 26.74;\ 30.97;\ 34.67;\ 41.04\}$ Pa.

The data were processed by the [3] method as follows. First, $\omega = f(\tau)$ polynomials were used to smooth the measurements. The equation $f(\hat{\tau}_0) = 0$ was solved to obtain the estimator $\hat{\tau}_0$.

Then points on the rheological curve τ_i and $\dot{\gamma}_i$ were calculated (in the ranges measured with the rheometer) and fitted to the corresponding models. Results: $\hat{\nu} = 5$, $\hat{\tau}_0 = 9.47$ Pa, $\hat{\mu} = 1.248 \cdot 10^{-2}$ Pa·sec, and $\hat{n} = 2$. Calculated shear stresses $\hat{\tau} = \{10.35; 15.96; 17.38; 18.63; 20.91; 24.68; 28.72; 32.09; 38.92\}$ Pa.

The standard deviation between the calculated and experimental values on the proposed method was 0.9%, as against 5.6% in the method based on estimating $\dot{\gamma}$ or 4.0% in the [3] method. The maximal errors on these methods were correspondingly 3.7, 9.2, and 32.5%. The methods based on approximating $\dot{\gamma}$ [1, 3] for high shear rates usually give systematic errors. The best interpretation performance in this range from [1, 3] is provided by the latter method. The [3] method differs from the proposed one in being very sensitive to the data volume and to the extrapolation to low shear rates.

The data were processed with the covariance matrix from experiment and by the [1, 3] methods (without the matrix), which showed that the differences in the estimators were more substantial, and sometimes there were errors in interpreting the model.

NOTATION

ω, angular velocity of outer cylinder; τ and a, shear stresses on the inner and outer cylinders; $\dot{\gamma}(\cdot)$, rheological model; $\dot{\gamma}$, shear rate gradient; p_ν, rheological-parameter vector, dimensions $1 \times m_\nu$; A, direct treatment operator (vector, dimensions $1 \times N$); n_ν, random-discrepancy vector, dimensions $1 \times N$; I, unit matrix, dimensions $N \times N$; P_ν, constraint on rheological-parameter vector; $2N_1$, number of quadrature nodes in segment $[a, \tau^j]$; A', derivative matrix, dimensions $m_\nu \times N$; A'*, matrix conjugate to A', dimensions $N \times m_\nu$; O, rheological-parameter estimator covariance matrix, dimensions $m_\nu \times m_\nu$.

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MEASUREMENT OF TEMPERATURE DISTRIBUTIONS IN A CONVECTIVE FLOW INDUCED BY POWERFUL OPTICAL RADIATION

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Results are presented from an experimental study of the convection which occurs within a liquid upon heating by a powerful light beam. The dependence of temperature on the convective jet axis upon power of the heating radiation is obtained.

When a powerful light beam propagates through an absorbing medium the latter heats up, its density changes, and as a result, convective flows develop. The low thresholds required for development of convection have attracted the attention of researchers to this phenomenon. Study of the mechanisms of photoabsorption convection may be of interest not only from the viewpoint of consideration of the processes accompanying propagation of powerful optical radiation through natural media (the atmosphere, ocean) but also as a new easily realized method for orienting flows in various industrial apparatus.

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